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Gaugino CP phases and EDMs in the extended gauge mediation SUSY breaking

Daijiro Suematsu and Hirokazu Tsuchida

Institute for Theoretical Physics, Kanazawa University,
Kanazawa 920-1192, Japan
E-mail: suematsu@hep.s.kanazawa-u.ac.jp, tsuchida@hep.s.kanazawa-u.ac.jp

Abstract: We study phenomenological aspects of the soft supersymmetry breaking parameters in a model with the extended gauge mediation supersymmetry breaking. In this model gaugino masses can be non-universal and as its result physical CP-phases remain in the gaugino sector even after the R-transformation. These phases contribute to the electric dipole moment (EDM) of an electron and a neutron. We show that their experimental bounds can be satisfied even for the situation such that there exist the order one CP-phases and the masses of superpartners are of the order of 100 GeV.

Keywords: Supersymmetry Phenomenology, Extended Supersymmetry, Supersymmetric Standard Model, CP violation.

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Contents

1. Introduction

At present low energy supersymmetry seems to be the most promising candidate for a solution of the gauge hierarchy problem or the weak scale stability. Although we have no direct evidence for the supersymmetry still now, the gauge coupling unification found in the minimal supersymmetric standard model (MSSM) may be considered as its indirect signal. When we consider the supersymmetric models, the supersymmetry breaking mechanism is crucial for their phenomenology. In fact, the experimental bounds on the flavor changing neutral currents (FCNC) severely restrict the supersymmetry breaking in the observable sector. They require the masses of the scalar superpartners to degenerate strictly. The gauge mediation supersymmetry breaking (GMSB) [1]–[6] is promising from this point of view because of its flavor blindness.

Another phenomenological constraint on the supersymmetry breaking comes from the electric dipole moment (EDM) of an electron and a neutron. It is well-known that the EDM of the electron and the neutron should be severely suppressed on the basis of the experimental data [7]. It has been recognized that there are two possibilities to satisfy this constraint [8]. One is that the soft breaking parameters are taken to be of the order of 100 GeV assuming that the soft CP-phases are smaller than 10^{-2}. Such small phases are usually considered to be unnatural and this aspect is considered as a default of the supersymmetric models. The other one is that the soft CP-phases are supposed to be of the order unity by assuming the soft scalar masses to take larger values than 1 TeV, which is considered to be unattractive from the view point of the weak scale supersymmetry. If we can have the third possibility in which the order one CP-phases and the superpartners with the masses of the order of 100 GeV can be consistent with the EDM constraints, it is very interesting and we can have a lot of interesting phenomenology [9]–[13]. In particular, if we consider the origin of the baryon number asymmetry in the universe, we may need new sources of CP violation. It is known that the Cabibbo-Kobayashi-Maskawa (CKM) phase in the standard model (SM) is insufficient to explain the baryon number asymmetry through the electroweak baryogenesis scenario because of a suppression due to the smallness of the quark flavor mixing. The possibility of the order one CP-phases in the soft supersymmetry breaking parameters seems to be fascinating since such soft CP-phases present us the promising sources for the CP-phases required in the electroweak baryogenesis and also they may allow us to relax the required Higgs mass bound [14].

When we consider this third possibility, it is useful to note that the usual analyses of the EDM are based on the assumption of the universal gaugino masses as stressed in [11]. If we loose this assumption, we can find the way out of the ordinary understanding. In fact, there are interesting suggestions that the experimental constraints on the EDM can be satisfied due to the effective cancellation among various contributions to the EDM[1] as far as the CP-phases in the gaugino masses are non-universal even in the case where both the order one CP-phases and rather light superpartners exist. Unfortunately, the non-universal gaugino masses seems to be rather difficult to be realized in both the unified theory and the superstring as discussed in [11, 16]. However, if we consider the GMSB such that some kind of discrete symmetry imposes the SU(3) triplet and SU(2) doublet messenger fields couple to the different singlet fields where the supersymmetry is broken due to the hidden sector dynamics, we can show that the non-universal phases in the gaugino masses appear naturally [17]. In the following discussion we will assume this extended structure in the messenger superpotential. In such a model the naturalness problem for the soft CP-phases may disappear since the EDM bounds for the electron and the neutron are satisfied by the cancellation among various contributions even for the order one CP-phases. Since the soft supersymmetry breaking parameters induced by the GMSB scenario is strongly constrained, we can survey such a possibility in the wide parameter region without large ambiguity.

This paper is organized as follows. In section 2 we discuss the soft supersymmetry breaking parameters in the extended GMSB. In section 3 we briefly review the formulas of the EDM of the electron and the neutron. In section 4 we discuss the feature of the soft supersymmetry breaking parameters using the numerical analysis based on the renormalization group equations. We also present our result for the estimation of the EDM in this model. The anomalous magnetic moment of a muon is predicted for these parameters. Section 5 is devoted to the summary. In the appendix we present an example for the realization of the required messenger superpotential.

[1]In the case of the electron EDM the cancellation between the chargino and neutralino contributions have been shown to occur [9, 10, 11]. On the other hand, in the case of the neutron EDM it has been known that there are several types of cancellation , that is, the cancellation between the gluino and the chargino exchange diagrams and also the cancellation among the gluino exchange diagrams in themselves *etc* [9, 15]. The combined effect of these cancellations allows the possibility of the large soft CP-phases [9, 10, 11].

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2. A model with the extended gauge mediation

In this section we introduce the extended GMSB and present the formulas for the supersymmetry breaking parameters in such a model. The messenger sector of the ordinary minimal GMSB model is defined by

$$W_{\rm min} = \lambda_q S q\bar{q} + \lambda_\ell S \ell\bar{\ell}\,, \tag{2.1}$$

where the messenger fields q ($\bar{q}$) and ℓ ($\bar{\ell}$) are the $\mathbf{3}(\bar{\mathbf{3}})$ of SU(3) and the $\mathbf{2}(\bar{\mathbf{2}})$ of SU(2), respectively. It is also assumed that (q, ℓ) can be embedded into the **5** of the grand unified group SU(5). If both of the scalar component S and the auxiliary F component of the gauge singlet superfield S get the vacuum expectation values $\langle S\rangle$ and $\langle F_S\rangle$ due to a suitable dynamics in the hidden sector, the gaugino masses and the soft scalar masses are generated at one-loop and two-loop level, respectively. If $\lambda_{q,\ell}^2\langle S\rangle^2 \gg \langle F_S\rangle$ is satisfied, their formulas are known to be the following simple forms by using $\Lambda = \langle F_S\rangle/\langle S\rangle$,

$$\begin{aligned} M_r &= c_r\frac{\alpha_r}{4\pi}\Lambda\,, \qquad \alpha_r = \frac{g_r^2}{4\pi}\,, \qquad c_3 = c_2 = \frac{3}{5}c_1 = 1\,, \\ \tilde{m}_f^2 &= 2|\Lambda|^2\left[C_3\left(\frac{\alpha_3}{4\pi}\right)^2 + C_2\left(\frac{\alpha_2}{4\pi}\right)^2 + \frac{5}{3}\left(\frac{Y}{2}\right)^2\left(\frac{\alpha_1}{4\pi}\right)^2\right], \end{aligned} \tag{2.2}$$

where $C_3 = 4/3$ and 0 for the SU(3) triplet and singlet fields, and $C_2 = 3/4$ and 0 for the SU(2) doublet and singlet fields, respectively. The hypercharge Y is expressed as $Y = 2(Q - T_3)$. The soft supersymmetry breaking parameters A_f and B for the scalar trilinear and bilinear terms are model dependent and cannot be directly related to the above formulas. If we take account of the effects of the radiative corrections, they can be written as [5]

$$\begin{aligned} A_f &\simeq A_f(\Lambda) + M_2(\Lambda)\left(-1.85 + 0.34|h_t|^2\right) + \cdots\,, \\ \frac{B}{\mu} &\simeq \frac{B}{\mu}(\Lambda) - A_t(\Lambda) + M_2(\Lambda)\left(-0.12 + 0.17|h_t|^2\right) + \cdots\,, \end{aligned} \tag{2.3}$$

where $A_f(\Lambda)$ and $B(\Lambda)$ are the initial values at which the supersymmetry breaking is introduced. In the expression of A_f the term with the top Yukawa coupling h_t should be neglected except for the top quark sector.

From these formulas we find that there cannot remain the physical CP-phases in the gaugino sector. In fact, even if the Λ is a complex, they can be rotated away by the R-transformation. Thus the physical CP-phases in the supersymmetry breaking parameters are confined in A_f and B. In the case of $A_f(\Lambda) = B(\Lambda) = 0$ which may be expected in many GMSB scenario, A_f and B are proportional to the gaugino mass and then the CP-phases in the soft supersymmetry breaking parameters are completely rotated away [3, 5].[2]

[2] Although this is an interesting solution for the soft CP phase problem at least in the case of the real μ, we do not take this possibility here.

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Now we consider to modify the superpotential $W_{\rm min}$ for the messenger fields. We assume that $(q,\ \bar{q})$ and $(\ell,\ \bar{\ell})$ couple with the different singlet chiral superfields $S_{1,2}$ due to some kind of symmetry [17]. Then the messenger superpotential takes the form such as

$$W_{\rm ext} = \lambda_q S_1 q\bar{q} + \lambda_\ell S_2 \ell\bar{\ell}\,. \tag{2.4}$$

If the singlet fields S_1 and S_2 couple with the hidden sector fields where the supersymmetry breaks down, $q, \bar{q}$ and $\ell, \bar{\ell}$ play the role of messenger fields as in the ordinary scenario. Only difference from the ordinary minimal GMSB scenario is that in the superpotential $W_{\rm ext}$ $q,\ \bar{q}$ and $\ell,\ \bar{\ell}$ couple with the different singlet chiral superfields. If we assume that both S_α and F_{S_α} get the VEVs due to the couplings with the supersymmetry breaking sector, the gaugino masses and the soft scalar masses are generated at one-loop and two-loop level, respectively, in the same way as the above mentioned ordinary case. However, the mass formulas are somewhat modified from the usual ones since the messenger fields $(q,\ \bar{q})$ and $(\ell,\ \bar{\ell})$ couple with the different singlets.

The gaugino masses can be written in the form as [17]

$$M_3 = \frac{\alpha_3}{4\pi}\Lambda_1\,, \qquad M_2 = \frac{\alpha_2}{4\pi}\Lambda_2\,, \qquad M_1 = \frac{\alpha_1}{4\pi}\left(\frac{2}{3}\Lambda_1 + \Lambda_2\right). \tag{2.5}$$

It is interesting that these formulas show that M_3 can be smaller than $M_{1,2}$ in the case of $|\Lambda_2| > |\Lambda_1|$. If we take account of the renormalization group evolution effect, their values at the weak scale M_W, for example, can be obtained as

$$M_r(M_W) = M_r(\Lambda)\frac{\alpha_r(M_W)}{\alpha_r(\Lambda)}\,, \tag{2.6}$$

where Λ is a scale at which the supersymmetry breaking is introduced. Since Λ_α is generally independent, the phases contained in the gaugino masses are non-universal even in the case of $|\Lambda_1| = |\Lambda_2|$. In that case we cannot remove them completely by using the R-transformation unlike the case of the universal gaugino mass. In fact, if we define the phases as $\Lambda_\alpha \equiv |\Lambda_\alpha|e^{i\theta_\alpha}$ and make M_2 real by the R-transformation, the phases of the gaugino masses M_r are written as

$$\begin{aligned} &\varphi_3 \equiv \arg(M_3) = \theta_1 - \theta_2\,, \qquad \varphi_2 \equiv \arg(M_2) = 0\,, \\ &\varphi_1 \equiv \arg(M_1) = \arctan\left(\frac{2|\Lambda_1|\sin(\theta_1-\theta_2)}{3|\Lambda_2| + 2|\Lambda_1|\cos(\theta_1-\theta_2)}\right). \end{aligned} \tag{2.7}$$

The scalar masses are induced through the two-loop diagrams as in the ordinary case. Their formulas can be written as

$$\tilde{m}_f^2 = 2|\Lambda_1|^2\left[C_3\left(\frac{\alpha_3}{4\pi}\right)^2 + \frac{2}{3}\left(\frac{Y}{2}\right)^2\left(\frac{\alpha_1}{4\pi}\right)^2\right] + 2|\Lambda_2|^2\left[C_2\left(\frac{\alpha_2}{4\pi}\right)^2 + \left(\frac{Y}{2}\right)^2\left(\frac{\alpha_1}{4\pi}\right)^2\right]. \tag{2.8}$$

If $|\Lambda_2| > |\Lambda_1|$ is realized, the SU(2) doublet fields tend to be heavy. As this result the color singlet fields can be heavier than the colored fields depending on the values of $\Lambda_{1,2}$. This seems to be a large difference from the ordinary scenario. As in the minimal GMSB

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model the soft supersymmetry breaking A_f and B parameters are model dependent also in this case. However, even in the case of $A_f(\Lambda) = B(\Lambda) = 0$ there can remain the physical CP-phases in the gaugino sector since the phases in the gaugino masses are not universal in general.

3. EDM and AMM of the leptons

We consider the MSSM with the soft supersymmetry breaking parameters which can be expressed by the mass formulas presented in the previous section. At first, we briefly review a relevant part of the MSSM to the study of the EDM of the quarks and leptons in order to fix the notation used here. Superpotential related to the lepton sector is given as

$$W = \sum_j \left(h_j^U Q_j H_2 \bar{U}_j + h_j^D Q_j H_1 \bar{D}_j + h_j^L L_j H_1 \bar{E}_j \right) + \mu H_1 H_2 \,, \tag{3.1}$$

where we take the basis in which the flavor mixings are resolved.[3] A supersymmetric mass parameter μ can be complex. The relevant soft supersymmetry breaking terms are introduced as

$$\begin{aligned} -\mathcal{L}_{\rm soft} = \sum_\alpha \tilde{m}_\alpha^2 |\phi_\alpha|^2 + \Bigg\{ \sum_j \Big(& A_j^U h_j^U \tilde{Q}_j H_2 \tilde{\bar{U}}_j + A_j^D h_j^D \tilde{Q}_j H_1 \tilde{\bar{D}}_j + A_j^L h_j^L \tilde{L}_j H_1 \tilde{\bar{E}}_j) + \\ & + B\mu H_1 H_2 + \frac{1}{2} \sum_r M_r \lambda_r \lambda_r + \text{h.c.} \Bigg\} \,, \end{aligned} \tag{3.2}$$

where we put a tilde for superpartners of the chiral superfields corresponding to the SM contents. The first term represents soft supersymmetry breaking masses for all scalar components of the chiral superfields. The third term in the parentheses represents the gaugino mass terms. Soft parameters B and A_j are the coefficients of the bilinear and trilinear scalar couplings and have a mass dimension. Although soft supersymmetry breaking parameters A_j, B and M_r can generally include the CP-phases, all of these are not independent physical phases. If we use the R-symmetry and redefine the fields appropriately, we can select out the physical CP-phases among them. We take them as

$$A_j = |A_j| e^{i\phi_{A_j}} \,, \qquad \mu = |\mu| e^{i\phi_\mu} \,, \qquad M_r = |M_r| e^{i\phi_r} \quad (r = 1, 3) \,, \tag{3.3}$$

where $B\mu$ and M_2 are real. These effective CP-phases are related to the original phases φ_i in the complex parameters introduced in eq. (2.7) as follows,

$$\phi_{A_j} = \varphi_{A_j} - \varphi_2 \,, \qquad \phi_\mu = -\varphi_B + \varphi_2 \,, \qquad \phi_{1,3} = \varphi_{1,3} - \varphi_2 \,. \tag{3.4}$$

It should be noted that in this definition the VEVs of the doublet Higgs scalars H_1 and H_2 are taken to be real.

[3] We do not consider a Yukawa coupling for neutrinos, for simplicity.

The mixing matrices in the sleptons, charginos and neutralinos are important elements to write down the formula for the EDM at the one-loop approximation. The mass terms of charginos can be written as

$$-\left(\tilde{H}_2^+, -i\lambda^+\right)\begin{pmatrix} |\mu|e^{i\phi_\mu} & \sqrt{2}m_Z c_W \sin\beta \\ \sqrt{2}m_Z c_W \cos\beta & M_2 \end{pmatrix}\begin{pmatrix} \tilde{H}_1^- \\ -i\lambda^- \end{pmatrix}, \tag{3.5}$$

where $\tan\beta = \langle H_2\rangle/\langle H_1\rangle$ and the abbreviations such as $s_W = \sin\theta_W$ and $c_W = \cos\theta_W$ are used. The mass eigenstates $\chi_i^\pm$ are defined in terms of the weak interaction eigenstates through the unitary transformations in such a way as

$$\begin{pmatrix} \chi_1^+ \\ \chi_2^+ \end{pmatrix} \equiv W^{(+)\dagger}\begin{pmatrix} \tilde{H}_2^+ \\ -i\lambda^+ \end{pmatrix}, \qquad \begin{pmatrix} \chi_1^- \\ \chi_2^- \end{pmatrix} \equiv W^{(-)\dagger}\begin{pmatrix} \tilde{H}_1^- \\ -i\lambda^- \end{pmatrix}. \tag{3.6}$$

Since we consider the GMSB and then the flavor mixing in the sfermion sector can be neglected, the sfermion mass matrices can be reduced into the 2×2 form for each flavor. This 2×2 sfermion mass matrix can be written in terms of the basis $(\tilde{f}_{L_\alpha}, \tilde{f}_{R_\alpha})$ as

$$\begin{pmatrix} |m_\alpha|^2 + \tilde{m}_{L_\alpha}^2 + D_{L_\alpha}^2 & m_\alpha(|A_\alpha|e^{i\phi_{A_\alpha}} + |\mu|e^{-i\phi_\mu}R_f) \\ m_\alpha(|A_\alpha|e^{-i\phi_{A_\alpha}} + |\mu|e^{i\phi_\mu}R_f) & |m_\alpha|^2 + \tilde{m}_{R_\alpha}^2 + D_{R_\alpha}^2 \end{pmatrix}, \tag{3.7}$$

where m_α and $\tilde{m}_{L_\alpha,R_\alpha}^2$ are the masses of the ordinary fermion f_α and its superpartners $\tilde{f}_{L_\alpha,R_\alpha}$, respectively. R_f is $\cot\beta$ for the up component of the SU(2) fundamental representation and $\tan\beta$ for the down component. $D_{L_\alpha}^2$ and $D_{R_\alpha}^2$ represent the D-term contributions, which are expressed as follows,

$$\begin{aligned} D_{L_\alpha}^2 &= m_Z^2\cos 2\beta(T_f^3 - Q_f s_W^2)\,, \\ D_{R_\alpha}^2 &= m_Z^2 s_W^2 Q_f \cos 2\beta\,, \end{aligned} \tag{3.8}$$

where T_f^3 takes 1/2 for the sfermions in the up sector and $-1/2$ for the ones in the down sector. Q_f is an electric charge of the field f. We define the mass eigenstates $(\tilde{f}_1, \tilde{f}_2)$ by the unitary transformation such as

$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} \equiv V^{\ell\dagger}\begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}. \tag{3.9}$$

If we take the canonically normalized neutralino basis as $\mathcal{N}^T = (-i\lambda_1, -i\lambda_2, \tilde{H}_1^0, \tilde{H}_2^0)$ and define their mass terms in such a form as $\mathcal{L}_{\rm mass}^{\rm n} = -\frac{1}{2}\mathcal{N}^T\mathcal{M}\mathcal{N} + {\rm h.c.}$, the 4×4 neutralino mass matrix $\mathcal{M}$ can be expressed as

$$\begin{pmatrix} |M_1|e^{i\phi_1} & 0 & -m_Z s_W\cos\beta & m_Z s_W \sin\beta \\ 0 & M_2 & m_Z c_W\cos\beta & -m_Z c_W\sin\beta \\ -m_Z s_W\cos\beta & m_Z c_W\cos\beta & 0 & -|\mu|e^{i\phi_\mu} \\ m_Z s_W\sin\beta & -m_Z c_W\sin\beta & -|\mu|e^{i\phi_\mu} & 0 \end{pmatrix}. \tag{3.10}$$

Mass eigenstates χ^0 of this mass matrix are related to $\mathcal{N}$ as

$$\chi^0 \equiv U^T\mathcal{N}\,, \tag{3.11}$$

where the mass eigenvalues are defined to be real and positive so that U includes Majorana phases.

Using these notations we give the formula for the EDM of the charged leptons. The effective interaction term representing the EDM of the charged lepton ℓ can be written as

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\, \mathcal{G}\; \bar{\ell}\sigma_{\mu\nu}\ell\; F^{\mu\nu}\,. \tag{3.12}$$

The value of the EDM of ℓ is related to this effective coupling $\mathcal{G}$ through the formula

$$d_\ell = \mathrm{Im}(\mathcal{G})\,. \tag{3.13}$$

In the MSSM there are new contributions to d_ℓ, which come from the one-loop diagram with the superpartners of the SM fields in the internal lines as is well-known [8, 9, 10]. These new contributions can be calculated as

$$d_\ell/e = -\frac{\alpha}{8\pi s_W^2} m_\ell \left(\frac{1}{m_\ell}\sum_{j,a}\frac{1}{m_j} G(x_{aj})\,\mathrm{Im}(A_{\chi_j^0}) + \frac{1}{m_W}\sum_{j}\frac{1}{m_j} F(x_{\tilde{\nu}j})\,\mathrm{Im}(A_{\chi_j^\pm}) \right), \tag{3.14}$$

where x_{aj} is defined as $x_{aj} = \tilde{m}_a^2/m_j^2$. m_j^2 is a mass eigenvalue of the chargino $\chi_j^\pm$ or the neutralino χ_j^0 and $\tilde{m}_a^2$ is a mass eigenvalue of the slepton $\tilde{f}_a$. In the right-hand side of eq. (3.14) the first term represents the neutralino-charged slepton contribution and the second term represents the chargino-sneutrino contribution. $A_{\chi_j^0}$ and $A_{\chi_j^\pm}$ which express the mixing factors appearing at each vertex are defined as

$$\begin{aligned} A_{\chi_j^0} &= -\Big[\left(U_{1j}^2 t_W^2 + U_{1j}U_{2j}t_W\right) V_{1a}^{\ell *} V_{2a}^{\ell} + \\ &\qquad + \frac{m_e}{2m_W\cos\beta}\left\{ (t_W U_{1j}U_{3j} + U_{2j}U_{3j})\,|V_{1a}^{\ell}|^2 - 2t_W U_{1j}U_{3j}|V_{2a}^{\ell}|^2 \right\}\Big]\,, \\ A_{\chi_j^\pm} &= \frac{1}{\sqrt{2}\cos\beta} W_{1j}^{(-)} W_{2j}^{(+)}\,, \end{aligned} \tag{3.15}$$

where $t_W = \tan\theta_W$ and we neglect a higher order term with respect to the charged lepton mass in $\mathcal{A}^0_{\chi_j}$. Since we have no right-handed neutrinos or they are considered to be decoupled in this expression, the slepton in the chargino contribution is fixed to be the left-handed sneutrino. Since the fermions in the external lines are very light compared with the sleptons in the internal lines, $G(x)$ and $F(x)$ are approximately written as

$$\begin{aligned} F(x) &= \frac{1-3x}{(1-x)^2} - \frac{2x^2}{(1-x)^3}\ln x\,, \\ G(x) &= \frac{1+x}{(1-x)^2} + \frac{2x}{(1-x)^3}\ln x\,. \end{aligned} \tag{3.16}$$

Here it is useful to note the following points to see the cancellation between the two contributions to the EDM. The chargino contribution is related to the CP-phase ϕ_μ although the neutralino contribution is caused by the CP-phases ϕ_1, ϕ_μ and ϕ_{A_α}. Generally the chargino contribution can be larger than the neutralino one because of the existence of the slepton mixing factor in the neutralino contribution. In order to make both contributions comparable the neutralino mass needs to be much lighter than the one of the chargino in addition to the existence of the suitable CP phases.

The real part of the same one-loop diagram as the ones for the EDM presents the anomalous magnetic moment (AMM). Thus it is also expressed by using the effective coupling $\mathcal{G}$ in eq. (3.12) as

$$a_\ell = \frac{2m_\ell}{e}\,\mathrm{Re}(\mathcal{G})\,. \tag{3.17}$$

We can calculate the new contribution to a_ℓ due to the superpartner effects in the same way as the EDM and the result is given by

$$\delta a_\ell = -\frac{\alpha}{4\pi s_W^2} m_\ell^2 \left(\frac{1}{m_\ell}\sum_{j,a}\frac{1}{m_j}G(x_{aj})\,\mathrm{Re}(A_{\chi_j^0}) + \frac{1}{m_W}\sum_j \frac{1}{m_j}F(x_{\tilde\nu j})\,\mathrm{Re}(A_{\chi_j^\pm})\right). \tag{3.18}$$

The value of the AMM of the muon is generally affected by the existence of the large CP-phases as it can be seen in eqs. (3.15) and (3.18). Its value can be largely changed from the ordinary estimation which is obtained under the assumption of no CP-phases if there are large CP-phases in the soft supersymmetry breaking parameters. Thus in our model the estimation of the AMM of the muon can be an interesting subject if the experimental bounds of the EDM of the electron and the neutron can be satisfied for the large CP-phases in the soft supersymmetry breaking parameters.

If we allow the nontrivial CP-phases in the gaugino masses as shown in eq. (3.3), the gluino mass can have a large CP-phase also. It can bring a large contribution to the EDM of the neutron in addition to the contributions due to the charginos and the neutralinos. We need to investigate them in order to check the consistency of the present model. In the estimation of the neutron EDM we use its nonrelativistic formula based on the quark EDM such as[4]

$$d_n = \frac{1}{3}\left\{(4d_d^g - d_u^g) + (4d_d^\chi - d_u^\chi)\right\}, \tag{3.19}$$

where d_u and d_d are the EDM of the u-quark and the d-quark and the superscripts g and χ represent the contributions from the one-loop diagrams containing the gluino internal line and the chargino/neutralino internal lines, respectively. This value of d_n should be evolved to the hadronic scale by including the QCD correction and then it gives $1.53d_n$ [9].

The gluino contribution to the quarks can be expressed as

$$\begin{aligned} d_f^g/e &= \frac{\alpha_s}{6\pi}\frac{Q_f}{|M_3|}\sum_{a=1}^{2}\mathrm{Im}(\mathcal{A}_g^{f_a})G(x_a)\,, \\ \mathcal{A}_g^{f_a} &= V_{2a}^f V_{1a}^{f*} e^{i\phi_3}\,, \end{aligned} \tag{3.20}$$

where $x_a = \tilde m_a^2/|M_3|^2$. In this contribution the related CP phases are ϕ_{A_α} and ϕ_μ in the squark mixing factors. This contribution can be small enough if the off-diagonal element of the squark mass matrix $|A_\alpha|e^{i\phi_{A_\alpha}} + |\mu|e^{-i\mu}R_f$ cancels by itself.

On the chargino and neutralino contributions to the quark EDM, we can calculate it as in the same way as the electron case. We find that it can be written as

$$d_u^\chi/e = \frac{-\alpha}{8\pi s_W^2} m_u \left[\frac{1}{m_u}\sum_{j,a}\frac{2}{3m_j}G(x_{aj})\,\mathrm{Im}(\mathcal{A}_{\chi_j^0}^{u_a}) + \right.$$

[4] In this analysis we do not consider the contribution to the neutron EDM from the chromoelectric and the CP-violating purely gluonic dimension six operators [9].

$$
+\frac{1}{m_W}\sum_{j,a}\frac{1}{m_j}\left\{F(x_{aj})-\frac{1}{3}G(x_{aj})\right\}\mathrm{Im}(\mathcal{A}^{u_a}_{\chi^\pm_j})\Bigg],
$$

$$
\begin{aligned}
d_d^{\chi}/e &= \frac{-\alpha}{8\pi s_W^2}m_d\Bigg[\frac{1}{m_d}\sum_{j,a}\frac{-1}{3m_j}G(x_{aj})\,\mathrm{Im}(\mathcal{A}^{d_a}_{\chi^0_j})+ \\
&\qquad +\frac{1}{m_W}\sum_{j,a}\frac{1}{m_j}\left\{-F(x_{aj})+\frac{2}{3}G(x_{aj})\right\}\mathrm{Im}(\mathcal{A}^{d_a}_{\chi^\pm_j})\Bigg],
\end{aligned}
\tag{3.21}
$$

where the mixing factors $\mathcal{A}^f_{\chi^\pm}$ and $\mathcal{A}^f_{\chi^0}$ are defined as

$$
\begin{aligned}
\mathcal{A}^{u_a}_{\chi^\pm_j} &= \frac{1}{\sqrt{2}\sin\beta}W^{(+)}_{1j}W^{(-)}_{2j}|V^d_{1a}|^2+\frac{1}{2\sin\beta\cos\beta}\frac{m_d}{m_W}W^{(+)}_{1j}W^{(-)}_{1j}V^{d*}_{2a}V^d_{1a}\,,\\
\mathcal{A}^{d_a}_{\chi^\pm_j} &= \frac{1}{\sqrt{2}\cos\beta}W^{(-)}_{1j}W^{(+)}_{2j}|V^u_{1a}|^2+\frac{1}{2\sin\beta\cos\beta}\frac{m_u}{m_W}W^{(+)}_{1j}W^{(-)}_{1j}V^{u*}_{2a}V^u_{1a}\,,\\
\mathcal{A}^{u_a}_{\chi^0_j} &= -\Bigg[\left(\frac{2}{9}t_W^2U^2_{1j}+\frac{2}{3}t_WU_{1j}U_{2j}\right)V^{u*}_{1a}V^u_{2a}-\frac{m_u}{2m_W\sin\beta}\times\\
&\qquad\times\left\{\left(\frac{1}{3}t_WU_{1j}U_{4j}+U_{2j}U_{4j}\right)|V^u_{1a}|^2-\frac{4}{3}t_WU_{1j}U_{4j}|V^u_{2a}|^2\right\}\Bigg]\\
\mathcal{A}^{d_a}_{\chi^0_j} &= -\Bigg[-\left(\frac{1}{9}t_W^2U^2_{1j}-\frac{1}{3}t_WU_{1j}U_{2j}\right)V^{d*}_{1a}V^d_{2a}-\frac{m_d}{2m_W\cos\beta}\times\\
&\qquad\times\left\{\left(\frac{1}{3}t_WU_{1j}U_{3j}-U_{2j}U_{3j}\right)|V^d_{1a}|^2+\frac{2}{3}t_WU_{1j}U_{3j}|V^d_{2a}|^2\right\}\Bigg],
\end{aligned}
\tag{3.22}
$$

where we again neglect the higher order terms of the quark mass in $\mathcal{A}^f_{\chi^0}$. The situation for the cancellation among these contributions is just the same as the electron case. The statement in the footnote 1 should be also reminded for the cancellation in the neutron EDM.

4. Numerical analysis

In this section we present the results of the numerical calculation of the masses of the superpartners, the EDM of the electron and the neutron and the AMM of the muon in our model. Before discussing the results of the caluculation we briefly explain our procedure for the numerical calculation.

We use the soft supersymmetry breaking parameters obtained in section 2 as the initial values at a suitable supersymmetry breaking scale and make them evolve to the weak scale by using the one-loop renormalization group equations (RGEs), except for the gauge and Yukawa couplings for which we use two-loop RGEs. Using the low energy parameters obtained in this way, we calculate the EDM of the electron and the neutron.

Free parameters related to the supersymmetry breaking are Λ_1 and Λ_2, which determine all of the masses of the gauginos and the scalar superpartners. Only the difference of their phases θ_1 and θ_2 is independent and it is related to the physical phases ϕ_3 and ϕ_1 defined in eq. (3.3) through eq. (2.7). Since μ, B, and A_α are dependent on the model and cannot be restricted in the present framework as stressed in the previous part, we do not fix their origin and we treat them as free parameters.[5] If we assume the universality of $A_\alpha(\Lambda)$ such as $A_\alpha(\Lambda) = A$, there are five independent real parameters ϕ_μ, ϕ_A, $|\mu|$, $|B|$, and $|A|$ where ϕ_B can be related to ϕ_μ by imposing the Higgs VEVs to be real under a phase convention such that $B\mu$ is real. Thus the soft supersymmetry breaking parameters are totally composed of 8 free real parameters in this study:

$$|\Lambda_1|\,, \qquad |\Lambda_2|\,, \qquad |A|\,, \qquad |B|\,, \qquad |\mu|\,, \qquad \tilde{\theta}\,, \qquad \phi_A\,, \qquad \phi_\mu\,, \tag{4.1}$$

where we define $\tilde{\theta} = \theta_1 - \theta_2$. There is an ambiguity on the scale where the soft supersymmetry breaking parameters are introduced and start running. In the present analysis we take $\Lambda = \max\,(|\Lambda_1|, |\Lambda_2|)$ as such a scale, for simplicity. This prescription is not expected to affect the results largely.[6]

In the region from the gauge coupling unification scale $M_{\rm U}$ to Λ the RGEs for the gauge coupling constants and Yukawa coupling constants are composed of the supersymmetric ones. The β-functions are calculated for the MSSM contents and the messenger fields. We solve these for various initial values of the Yukawa couplings. At Λ the messenger fields are supposed to decouple and the soft supersymmetry breaking parameters are introduced. Thus the RGEs become the same as the ones of the MSSM. We can obtain their values at the electroweak scale by solving these RGEs numerically. In order to determine the phenomenologically interesting parameter region, we impose several conditions on the weak scale parameters obtained by the RGEs. As such conditions we adopt the following ones:

(i) The physical true vacuum should be radiatively realized as the minimum of the tree-level scalar potential in the consistent way with the masses of the top, bottom quarks and the tau lepton. We check the consistency between the values of $\tan\beta$ predicted from these two different physical requirement.

(ii) Various experimental mass bounds for the superpartners, such as gluinos, charginos, stop, stau, and the charged Higgs scalar should be satisfied.[7] The color and electromagnetic charge should not be broken.

After restricting the parameter space at the high energy scale by imposing these conditions on the weak scale values, we finally calculate the EDM of the electron and the neutron and impose their experimental bounds [7, 19]

$$\left|\frac{d_e}{e}\right| = 1.6 \times 10^{-27}\,\mathrm{cm}\,, \qquad \left|\frac{d_n}{e}\right| = 1.2 \times 10^{-25}\,\mathrm{cm}\,,$$

to restrict the selected parameter region further.

[5] If we assume $A_\alpha(\Lambda) = B(\Lambda) = 0$ in eq. (2.3), they can be definitely determined through the radiative effect by using $\Lambda_{1,2}$. However, we do not adopt this possibility but treat them in more general way.

[6] Since we mainly study the region where $|\Lambda_2|/|\Lambda_1|$ is not so large, this treatment will be justified.

[7] In this study the large CP-phases are assumed to exist. Since the CP even neutral Higgs scalars mix with the CP odd neutral Higgs scalar, the Higgs mass formulas are changed [18]. Thus we do not impose the neutral Higgs mass bound.

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In the following part we focus our stusy into the cases with the large physical CP-phases such as $\phi_A = \pi/2$ and $\tilde{\theta} = 3\pi/2$. Although we search the possible region of $\Lambda_{1,2}$ and ϕ_μ, we restrict our study for the parameters $|A|$, $|B|$ and $|\mu|$ into 100 GeV $\leq |A|,\ |B|,\ |\mu| \leq$ 500 GeV.[8]

4.1 Spectrum of superpartners

In the present model the mass parameters of the superpartners are represented by the restricted number of input parameters as shown in the previous part. Their masses are all determined only by $\Lambda_{1,2}$. The mass parameters of the superpartners are also related to the realization of the radiative symmetry breaking at the weak scale. This fact makes intimately relate μ and B to $\Lambda_{1,2}$ in the present model. The minimum conditions for the tree-level scalar potential can be written as

$$\sin 2\beta = \frac{2B\mu}{m_1^2 + m_2^2 + 2|\mu|^2}, \qquad m_Z^2 = -2|\mu|^2 + \frac{2m_1^2 - 2m_2^2 \tan^2\beta}{\tan^2\beta - 1}. \tag{4.2}$$

These conditions tell us what kind of tuning for the μ and B parameters are required to realize the correct vacuum in the present supersymmetry breaking scenario [20]. Since we have no knowledge for μ and B in the present scenario, the study of these conditions seems to give us a useful information for them. In figure 1 we show what kind of values of μ and B are required as a function of $|\Lambda_2|/|\Lambda_1|$. This figure shows that we can have the most solutions in the region of $|B| > |\mu|$ for $|\Lambda_2|/|\Lambda_1| \gtrsim 1.7$, although there are solutions only in the region of $|B| < |\mu|$ for $|\Lambda_2|/|\Lambda_1| \simeq 1$ which corresponds to the ordinary minimal GMSB [21]. This shows that the present model can relax the condition for the relative magnitude of $|\mu|$ and $|B|$ which is required by the radiative symmetry breaking. Since the value of $\tan\beta$ tends to be small in the case of $|\Lambda_2| > |\Lambda_1|$ as discussed in [21], we restrict the $\tan\beta$ value into the favored region such as $2.7 \lesssim \tan\beta \lesssim 3.2$ and present the results.

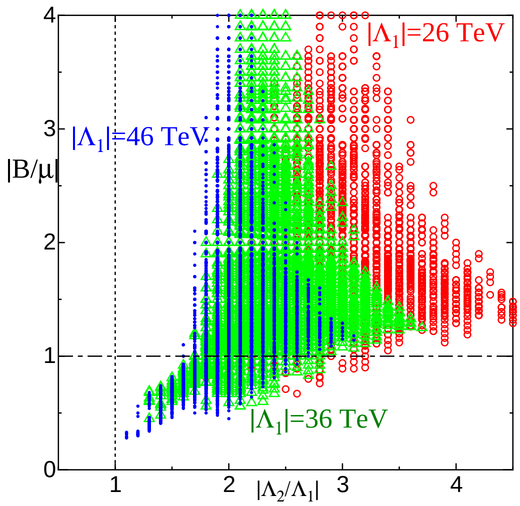

Figure 1: The ratio of $|B|$ to $|\mu|$ required by the radiative symmetry breaking conditions.

As stressed before, the mass formulas of the superpartners can be written only by two parameters $\Lambda_{1,2}$ and then the model is very predictive at least for the masses of superpartners. It is interesting that the spectrum can be largely different from the ones of the ordinary GMSB scenario. In order to display the feature of the mass spectrum of superpartners, it is convenient to plot them as the functions of $|\Lambda_2|/|\Lambda_1|$. We give them in figure 2. As is easily seen from these figures, this scenario predicts that the next lightest

[8] We also study the case of $A = 0$, in which the free parameter can be reduced into six. The result is not much different from this case and it can be included in the region given here.

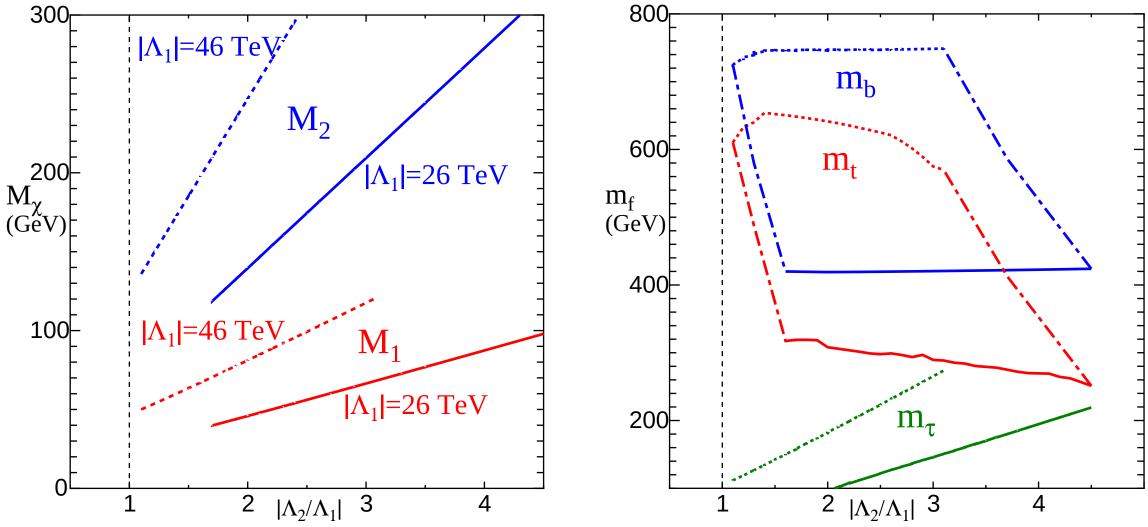

Figure 2: Mass spectrum of the low lying superpartners for various values of $|\Lambda_{1,2}|$. The left panel shows the gaugino masses related to charginos and neutralinos. The right panel shows the third generation sfermion masses. In each panel the solid lines correpsond to $|\Lambda_1| = 26$ TeV and the dotted lines correspond to $|\Lambda_1| = 46$ TeV.

superparticle (NLSP) can be a neutralino. This is rather different feature from the ordinary GMSB where the NLSP tends to be the right-handed stau. The right-handed stop is rather light in the $|\Lambda_2|/|\Lambda_1| > 1$ region. Since the right-handed stop has the contribution only through the $\mathrm{U}(1)_Y$ coupling at $|\Lambda_2|$ and also the gluino mass which is determined by $|\Lambda_1|$ is small, the RGE evolution can reduce the right-handed stop mass in the case of the large $|\Lambda_2|/|\Lambda_1|$. These are related to the general feature of this model such that the SU(2) nonsinglet fields tend to be heavier than the SU(2) singlet fields. This type of spectrum of superpartners can make the unification scale of the gauge coupling constants higher than the ordinary one of the MSSM.[9]

4.2 EDMs of the electron and the neutron

In the previous section we have discussed what kind of contributions for the EDM can exist as the effects of the superpartners. Here we show two important ingredients for the cancellations between various contributions to the EDM by using the numerical analysis. In the left panel of figure 3 we plot the allowed upper and lower bound values of $|\Lambda_2|$ as a function of $|\Lambda_1|$ by imposing the phenomenological constraints. This shows that we can obtain the solutions only in the $|\Lambda_1| < |\Lambda_2|$ region. If we do not impose the EDM constraints, we can find the solutions also in the region $|\Lambda_1| > |\Lambda_2|$ [21]. Thus this result is considered to be caused by the EDM constraints. In this region the neutralino can be much lighter than the chargino, which is expected from the superpartners mass formulas. (See figure 2 also.) This aspect of the mass spectrum seems to make the neutralino contribution

[9] This possibility has been discussed in the different context in [22]. Our present model can realize such a spectrum in a natural way.

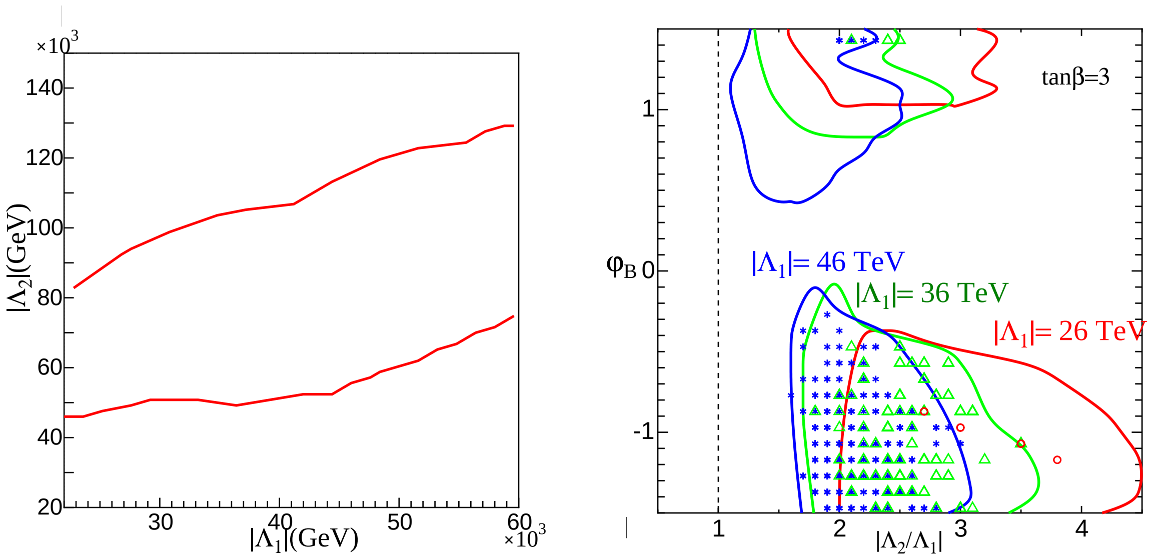

Figure 3: The left panel shows the allowed region in the ($|\Lambda_1|$, $|\Lambda_2|$) plane which satisfies both the EDM constraints and the phenomenological conditions given in the text. The right panel shows the allowed region of $\phi_B(= -\phi_\mu)$ for the same constraints. In both panel the region surrounded by the curves are allowed.

larger and then the cancellations between the chargino contribution and the neutralino contribution to the EDM is considered to be effective.

On the CP-phases we have discussed how each phase contributes to the EDM in the previous section. In this numerical study we assume the maximum values for ϕ_A and $\tilde{\theta}$. The large $\tilde{\theta}$ results in the large CP-phases in the gaugino sector. In the right panel of figure 3 we show the required value for ϕ_μ to satisfy the EDM constraints. We can find that the rather large value of ϕ_μ is required for the cancellations of the various contributions to the EDM of the electron and the neutron, as is expected from the previous discussion. This result shows that the large CP-phases in the soft supersymmetry breaking parameters can be consistent with the EDM constraints of the electron and the neutron as far as the large CP-phases exist in the gaugino sector.

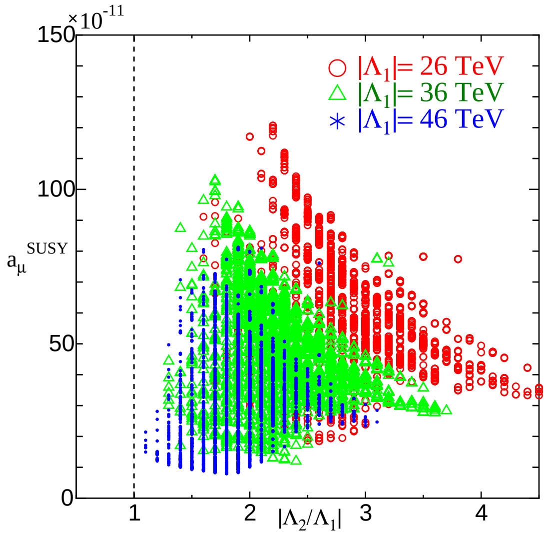

Figure 4: The predicted value of the AMM of a muon in the parameter space where the EDM bounds and the phenomenological constraints are satisfied.

It is interesting that our model can realize the convenient situation for the cancellation for the EDM of the electron and the neutron, that is, the desirable mass spectrum for the case of $|\Lambda_2| > |\Lambda_1|$ and also the presence of physical CP-phases in the gaugino sector. The predicted lower bounds for the

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electron EDM and the neutron EDM are

$$\left|\frac{d_e}{e}\right| \gtrsim 10^{-31\sim-30}\,\mathrm{cm}\,, \qquad \left|\frac{d_n}{e}\right| \gtrsim 10^{-29\sim-28}\,\mathrm{cm}\,.$$

We also show the predicted value for the AMM of the muon in figure 4. These values are less than the half of the value of the difference between the experimental value and the SM prediction, $a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = (345 \pm 114) \times 10^{-11}$, which is presented in [23].

Finally we should comment on the constraint from the EDM of the mercury atom. It has been pointed out that the limit from the mercury EDM is stronger than the ones coming from the neutron or electron EDM [24]. Their work shows that the parameter region where the cancellation occurs among various contributions to the EDM is different between the mercury and the electron. However, that study has been done in the MSSM with the universal gaugino masses. The situation can be different in our case where the gaugino masses are non-universal and there are additional CP-phases. We have calculated the mercury EDM in the allowed parameter region obtained in the present study of the neutron and electron EDMs. The parameters which satisfy the constraint on the color EDMs of quarks from the mercury EDM [24] are shown in the right panel of figure 3 by three types of simbols. They correspond to the ones shown in figure 4. This figure shows that the allowed parameter space shrinks by adding the mercury constraint but there remain nonnegligible regions. We will present the detailed results elsewhere [25].

5. Summary

We have investigated the non-universal CP-phases in the gaugino masses and their effects on the EDM constraints in the extended GMSB. We have shown that the non-universal gaugino masses can be generally realized if we assume that the SU(3) triplet and the SU(2) doublet messenger fields couple to the different singlet chiral superfields which are assumed to break the supersymmetry through the hidden sector dynamics. As such an example we have presented a model with the direct product gauge structure $\mathrm{SU}(5)'\times\mathrm{SU}(5)''$ in the appendix. In this model the discrete symmetry introduced to realize the doublet-triplet splitting simultaneously forces the SU(3) triplet and the SU(2) doublet messenger fields to couple to the different singlet chiral superfields.

In this type of model the characteristic spectrum of superpartners is induced and also the non-universal CP-phases in the gaugino masses can be introduced. The SU(2) nonsinglet superpartners tend to be heavier than the SU(2) singlet ones. The CP-phases can remain in the gaugino sector as the physical ones after the R-transformation. These may result in the various interesting phenomenology different from ordinary GMSB scenario. As the most interesting one, we have calculated the effect of the CP-phases on the EDM of the electron and the neutron by solving the RGEs for the soft supersymmetry breaking parameters obtained in our model. We have found that the experimental bounds can be satisfied since the effective cancellation occurs between the neutralino and chargino contributions even for the order one CP-phases without assuming the heavy superpartners of the $O(1)$ TeV masses. It is interesting that the mercury EDM constraint is also satisfied

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in the same parameter region, although the allowed region shrinks. The cancellation is considered to be caused mainly by the existence of the CP-phases in the gaugino sector and the feature of the mass spectrum such that the neutralino can be much lighter than the chargino. The same origin for them also makes the right-handed stop rather light and also the neutralino lighter than the right-handed stau. Since the SU(2) nonsinglet superpartners may decouple earlier than others, the gauge coupling unification can be realized at the higher scale in comparison with the MSSM.

Further phenomenological study of the model seems to be necessary since the essential feature of the model may be related to the reasonable motivation to solve the doublet-triplet splitting problem [26] in the grand unified model. In particular, the Higgs sector can be affected by the the existence of the large CP-phases in the soft supersymmetry breaking parameters [27]. Since the CP even Higgs scalar field can be mixed with the CP odd Higgs scalar, the lightest neutral Higgs mass can be largely modified from the one of the ordinary case. This aspect of the present model will be studied elsewhere.

Acknowledgments

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A. A realization of the extended GMSB model

In this appendix we give an example which can realize the extended GMSB [17]. The model is defined by the direct product gauge structure such as $\mathcal{G}$ =SU(5)$'\times$SU(5)$''$ and a global discrete symmetry F which commutes with this gauge symmetry $\mathcal{G}$. We introduce the chiral superfields summarized in table 1. In order to cause the symmetry breaking at the high energy scale the $\mathcal{G}\times F$ invariant renormalizable superpotential for Σ, Φ_1 and Φ_2 is assumed as

$$W_1 = M_\phi \,\mathrm{Tr}\,(\Phi_1\Phi_2) + \frac{1}{2}M_\sigma \,\mathrm{Tr}\,(\Sigma^2) + \lambda\,\mathrm{Tr}\left(\Phi_1\Sigma\Phi_2 + \frac{1}{3}\Sigma^3\right), \tag{A.1}$$

The scalar potential induced from this W_1 can be obtained as

$$V = \mathrm{Tr}\,|M\phi_1 + \lambda\phi_1\sigma + y|^2 + \mathrm{Tr}\,|M\phi_2 + \lambda\sigma\phi_2 + x|^2 + \mathrm{Tr}\,|M\sigma + \lambda\phi_1\phi_2 + \sigma^2 + z|^2\,, \tag{A.2}$$

where $\phi_{1,2}$ and σ are the scalar components of $\Phi_{1,2}$ and Σ, respectively. They are traceless and x, y and z are the Lagrange multipliers for these traceless conditions. We can easily find a non-trivial solution of the V minimum such as

$$\phi_2 = \frac{x}{y}\phi_1\,, \tag{A.3}$$

$$M_\phi\phi_1 + \lambda\phi_1\sigma + y = 0\,, \tag{A.4}$$

$$M_\sigma\sigma + \lambda\left(\sigma^2 + \frac{x}{y}\phi_1^2\right) + z = 0\,, \tag{A.5}$$

	$\mathcal{F}(\mathcal{G}$ rep.$)$	F	F'	
			$\mathbf{3} \in \mathbf{5}$ or $\bar{\mathbf{3}} \in \bar{\mathbf{5}}$	$\mathbf{2} \in \mathbf{5}$ or $\bar{\mathbf{2}} \in \bar{\mathbf{5}}$
Quarks/Leptons	$\Psi^j_{10}(\mathbf{10},\mathbf{1})$	α	α	α
$(j = 1 \sim 3)$	$\Psi^j_{\bar{5}}(\bar{\mathbf{5}},\mathbf{1})$	β	β	β
Higgs fields	$H(\mathbf{5},\mathbf{1})$	γ	γ	γ
	$\tilde{H}(\mathbf{1},\bar{\mathbf{5}})$	ξ	$\xi + 2a$	$\xi - 3a$
Messenger fields	$\bar{\chi}(\bar{\mathbf{5}},\mathbf{1})$	δ	δ	δ
	$\chi(\mathbf{1},\mathbf{5})$	ζ	$\zeta - 2a$	$\zeta + 3a$
Bifundamental field	$\Phi_1(\bar{\mathbf{5}},\mathbf{5})$	η	$\eta + 2b$	$\eta - 3b$
	$\Phi_2(\mathbf{5},\bar{\mathbf{5}})$	σ	$\sigma - 2b$	$\sigma + 3b$
Adjoint Higgs field	$\Sigma(\mathbf{1},\mathbf{24})$	0	0 (for Σ^i_i)	
Singlets	$S_1(\mathbf{1},\mathbf{1})$	θ	θ	
	$S_2(\mathbf{1},\mathbf{1})$	τ	τ	

Table 1: Discrete charge assignment for the chiral superfields. For the adjoint Higgs field Σ we only give the charge for the diagonal components.

where the Lagrange multipliers y and z are determined as

$$y = -\frac{\lambda}{5}\,\mathrm{Tr}\,(\phi_1\sigma)\,, \qquad z = -\frac{\lambda}{5}\,\mathrm{Tr}\left(\sigma^2 - \frac{5x}{\lambda\,\mathrm{Tr}(\phi_1\sigma)}\right), \tag{A.6}$$

and x remains as a free parameter. If we restrict ourselves to a special direction in the field space such as $\phi_1 = \kappa\sigma$ and also assume $M_\sigma = M_\phi(1 + x\kappa^2/y)$, eqs. (A.4) and (A.5) are reduced into the same equation for the adjoint Higgs scalar in the ordinary supersymmetric SU(5) model as

$$M_\phi\sigma + \lambda\sigma^2 - \frac{\lambda}{5}\,\mathrm{Tr}\left(\sigma^2\right) = 0\,. \tag{A.7}$$

We adopt the most interesting one among three degenerate independent solutions, which can be written as

$$\sigma = \tilde{M}\ \mathrm{diag}\ (2,\, 2,\, 2,\, -3,\, -3)\,, \tag{A.8}$$

where $\tilde{M}$ is defined as $\tilde{M} = M_\phi/\lambda$. Using this σ, other fields are determined as

$$\phi_1 = \kappa\sigma\,, \qquad \phi_2 = \frac{1}{\kappa}\left(\frac{M_\sigma}{M_\phi} - 1\right)\sigma\,, \tag{A.9}$$

where κ is an undetermined parameter.

The vacuum defined by eqs. (A.8) and (A.9) is found to be invariant under the gauge transformation of $\mathcal{H}$ =SU(3)×S(2)×U(1) which is the subgroup of the diagonal sum SU(5) of $\mathcal{G}$. If we assume that the model is based on the deconstruction method [29], the remaining discrete symmetry is $F' = F \times G_{\mathrm{U}(1)''}$ where $G_{\mathrm{U}(1)''}$ is the discrete subgroup of the hypercharge U(1)$''$ of SU(5)$''$ [28]. The charge assignment of F' is shown in table 1.[10]

[10] We assume that SU(5)$''$ is induced as the diagonal sum of two SU(5) effectively and also $\tilde{H}, \chi$ and Φ_1, Φ_2 belong to the different SU(5), respectively. This assumption makes it possible to introduce the independent charge normalization a and b for $G_{\mathrm{U}(1)''}$. Since $G_{\mathrm{U}(1)''}$ is not just U(1)$''$ but its discrete subgroup Z_n, its charges of $\tilde{H}$, χ and $\Phi_{1,2}$ can be taken independently and the assumption for SU(5)$''$ may not be ncessary.

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After the symmetry breaking from $\mathcal{G} \times F$ into $\mathcal{H} \times F'$, superpotential structure is expected to be changed into the $\mathcal{H} \times F'$ invariant one as in the case of the Wilson line breaking in the heterotic string.

We require various conditions on F and F' to satisfy phenomenological constraints to realize our purpose. As such conditions we take the following ones.

(i) Each term in the F invariant superpotential W_1 should exist before the symmetry breaking and this requirement imposes the condition

$$\eta + \sigma = 0\,. \tag{A.10}$$

(ii) The gauge invariant bare mass terms of the fields such as $\Psi_{\bar{5}} H$, $H\bar{\chi}$, $\tilde{H}\chi$ and $S_\alpha S_\beta$ should be forbidden by both F and F'. These conditions are summarized as

$$\begin{aligned} &\beta + \gamma \neq 0\,, \qquad \gamma + \delta \neq 0\,, \qquad \xi + \zeta \neq 0\,, \\ &2\theta \neq 0\,, \qquad 2\tau \neq 0\,, \qquad \theta + \tau \neq 0\,. \end{aligned} \tag{A.11}$$

(iii) To realize the doublet-triplet splitting [26] Yukawa coupling $\Phi_1 H\tilde{H}$ should be forbidden by F. Moreover, $\Phi_{1,2} H_2 \tilde{H}_2$ and $\Sigma H_2 \tilde{H}_2$ should also be forbidden by F' after the symmetry breaking, although $\Phi_1 H_3 \tilde{H}_3$ is allowed at least. This gives the conditions such as

$$\begin{aligned} &\gamma + \xi + \eta \neq 0\,, \qquad \gamma + \xi + \eta + 2(a+b) = 0\,, \\ &\gamma + \xi + \eta - 3(a+b) \neq 0\,, \qquad \gamma + \xi + \sigma - 3(a-b) \neq 0\,, \qquad \gamma + \xi - 3a \neq 0\,. \end{aligned} \tag{A.12}$$

(iv) Yukawa couplings of quarks and leptons, that is, $\Psi_{10}\Psi_{10}H_2$ and $\Psi_{10}\Psi_{\bar{5}}\tilde{H}_{\bar{2}}\Phi_1$ should exist at least under F'. This requires

$$2\alpha + \gamma = 0\,, \qquad \alpha + \beta + \xi + \eta - 3(a+b) = 0\,. \tag{A.13}$$

(v) The chiral superfields χ and $\bar{\chi}$ should be massless at the $\mathcal{G}$ breaking scale due to F and they play the role of the messenger fields of the supersymmetry breaking which is assumed to occur in the S_α sector. These require the absence of $\Phi_2\chi\bar{\chi}$ under F and also the absence of $\Phi_{1,2}\chi\bar{\chi}$ and $\Sigma\chi\bar{\chi}$ under F', although the existence of $\Phi_2 S_\alpha \chi\bar{\chi}$ under F' is needed. These conditions can be written as

$$\begin{aligned} &\delta + \zeta + \sigma \neq 0\,, \quad \delta + \zeta + \sigma - 2(a+b) + \theta = 0\,, \quad \delta + \zeta + \sigma + 3(a+b) + \tau = 0\,, \\ &\delta + \zeta - 2a \neq 0\,, \quad \delta + \zeta + \sigma - 2(a+b) \neq 0\,, \quad \delta + \zeta + \eta - 2(a-b) \neq 0\,, \\ &\delta + \zeta + 3a \neq 0\,, \quad \delta + \zeta + \sigma + 3(a+b) \neq 0\,, \quad \delta + \zeta + \eta + 3(a-b) \neq 0\,. \end{aligned} \tag{A.14}$$

(vi) The neutrino should be massive and the proton should be stable.[11] This means that $\Phi_{\bar{5}}^2 H_2^2$ should exist and $\Psi_{10}\Psi_{\bar{5}}^2$ and $\Psi_{10}^3\Psi_{\bar{5}}$ should be forbidden. These require

$$2(\beta + \gamma) = 0\,, \qquad \alpha + 2\beta \neq 0\,, \qquad 3\alpha + \beta \neq 0\,. \tag{A.15}$$

All of these conditions should be understood up to the modulus n when we take $F' = Z_n$.

[11] The magnitude of the neutrino masses realized in this way depends on the details of the model and we do not discuss this point further here.

We can easily find an example of the consistent solution for these constraints. For example, if we take $F' = Z_{20}$, such an example can be given as

$$\begin{aligned} &\alpha = \eta = \zeta = -\sigma = a = 1\,, \qquad \gamma = -b = -2\,, \\ &\delta = \theta = 3\,, \qquad \xi = -5\,, \qquad \beta = -\tau = -8\,. \end{aligned} \tag{A.16}$$

It should be noted that the existence of the different singlet fields $S_{1,2}$ is generally required in order to make χ and $\bar{\chi}$ play a role of messengers of the supersymmetry breaking. In fact, the F' charges of χ and $\bar{\chi}$ satisfy

$$\theta - \tau = 5(a+b) \neq 0\,, \quad (\mathrm{mod}\ n) \tag{A.17}$$

which is derived from eq. (A.14). This feature is caused by the discrete symmetry which is related to the doublet-triplet splitting.

References

[1] M. Dine, W. Fischler and M. Srednicki, *Supersymmetric technicolor*, *Nucl. Phys.* **B 189** (1981) 575;
S. Dimopoulos and S. Raby, *Supercolor*, *Nucl. Phys.* **B 192** (1981) 353;
M. Dine and W. Fischler, *A phenomenological model of particle physics based on supersymmetry*, *Phys. Lett.* **B 110** (1982) 227;
M. Dine and M. Srednicki, *More supersymmetric technicolor*, *Nucl. Phys.* **B 202** (1982) 238;
M. Dine and W. Fischler, *A supersymmetric GUT*, *Nucl. Phys.* **B 204** (1982) 346;
L. Alvarez-Gaumé, M. Claudson and M.B. Wise, *Low-energy supersymmetry*, *Nucl. Phys.* **B 207** (1982) 96;
C.R. Nappi and B.A. Ovrut, *Supersymmetric extension of the* SU(3) × SU(2) × U(1) *model*, *Phys. Lett.* **B 113** (1982) 175;
S. Dimopoulos and S. Raby, *Geometric hierarchy*, *Nucl. Phys.* **B 219** (1983) 479.

[2] M. Dine, A.E. Nelson and Y. Shirman, *Low-energy dynamical supersymmetry breaking simplified*, *Phys. Rev.* **D 51** (1995) 1362 [hep-ph/9408384];
M. Dine, A.E. Nelson, Y. Nir and Y. Shirman, *New tools for low-energy dynamical supersymmetry breaking*, *Phys. Rev.* **D 53** (1996) 2658 [hep-ph/9507378].

[3] M. Dine, Y. Nir and Y. Shirman, *Variations on minimal gauge mediated supersymmetry breaking*, *Phys. Rev.* **D 55** (1997) 1501 [hep-ph/9607397].

[4] S. Dimopoulos, M. Dine, S. Raby and S. Thomas, *Experimental signatures of low energy gauge mediated supersymmetry breaking*, *Phys. Rev. Lett.* **76** (1996) 3494 [hep-ph/9601367].

[5] K.S. Babu, C.F. Kolda and F. Wilczek, *Experimental consequences of a minimal messenger model for supersymmetry breaking*, *Phys. Rev. Lett.* **77** (1996) 3070 [hep-ph/9605408].

[6] G.F. Giudice and R. Rattazzi, *Theories with gauge-mediated supersymmetry breaking*, *Phys. Rept.* **322** (1999) 419 [hep-ph/9801271];
T. Ibrahim and P. Nath, *CP-violation in SUSY, strings and branes*, hep-ph/0207213.

[7] B.C. Regan, E.D. Commins, C.J. Schmidt and D. DeMille, *New limit on the electron electric dipole moment*, *Phys. Rev. Lett.* **85** (2002) 071805.

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[8] J.R. Ellis, S. Ferrara and D.V. Nanopoulos, *CP-violation and supersymmetry*, *Phys. Lett.* **B 114** (1982) 231;
W. Buchmuller and D. Wyler, *CP-violation and r invariance in supersymmetric models of strong and electroweak interactions*, *Phys. Lett.* **B 121** (1983) 321;
J. Polchinski and M.B. Wise, *The electric dipole moment of the neutron in low-energy supergravity*, *Phys. Lett.* **B 125** (1983) 393;
M. Dugan, B. Grinstein and L.J. Hall, *CP-violation in the minimal $N = 1$ supergravity theory*, *Nucl. Phys.* **B 255** (1985) 413;
Y. Kizukuri and N. Oshimo, *The neutron and electron electric dipole moments in supersymmetric theories*, *Phys. Rev.* **D 46** (1992) 3025.

[9] T. Ibrahim and P. Nath, *The chromoelectric and purely gluonic operator contributions to the neutron electric dipole moment in $N = 1$ supergravity*, *Phys. Lett.* **B 418** (1998) 98 [hep-ph/9707409]; *The neutron and the electron electric dipole moment in $N = 1$ supergravity unification*, *Phys. Rev.* **D 57** (1998) 478 [hep-ph/9708456].

[10] M. Brhlik, G.J. Good and G.L. Kane, *Electric dipole moments do not require the CP-violating phases of supersymmetry to be small*, *Phys. Rev.* **D 59** (1999) 115004 [hep-ph/9810457];
R. Arnowitt, B. Dutta and Y. Santoso, *SUSY phases, the electron electric dipole moment and the muon magnetic moment*, *Phys. Rev.* **D 64** (2001) 113010 [hep-ph/0106089].

[11] M. Brhlik, L.L. Everett, G.L. Kane and J. Lykken, *A resolution to the supersymmetric CP problem with large soft phases via D-branes*, *Phys. Rev. Lett.* **83** (1999) 2124 [hep-ph/9905215]; *Superstring theory and CP-violating phases: can they be related?*, *Phys. Rev.* **D 62** (2000) 035005 [hep-ph/9908326].

[12] M. Brhlik, L.L. Everett, G.L. Kane, S.F. King and O. Lebedev, *Can supersymmetry soft phases be the source of all CP-violation?*, *Phys. Rev. Lett.* **84** (2000) 3041 [hep-ph/9909480].

[13] M. Brhlik, L.L. Everett, G.L. Kane, S.F. King and O. Lebedev, *Can supersymmetry soft phases be the source of all CP-violation?*, *Phys. Rev. Lett.* **84** (2000) 3041 [hep-ph/9909480].

[14] M. Brhlik, G.J. Good and G.L. Kane, *Effects of large CP-violating soft phases on supersymmetric electroweak baryogenesis*, *Phys. Rev.* **D 63** (2001) 035002 [hep-ph/9911243].

[15] T. Kobayashi, M. Konmura, D. Suematsu, K. Yamada and Y. Yamagishi, *Neutron electric dipole moment under nonuniversal soft SUSY breaking terms.*, *Prog. Theor. Phys.* **94** (1995) 417.

[16] G.L. Kane, J. Lykken, B.D. Nelson and L.-T. Wang, *Re-examination of electroweak symmetry breaking in supersymmetry and implications for light superpartners*, *Phys. Lett.* **B 551** (2003) 146 [hep-ph/0207168].

[17] D. Suematsu, *Soft SUSY breaking masses in a unified model with doublet- triplet splitting*, *Phys. Rev.* **D 67** (2003) 075020 [hep-ph/0212244].

[18] M. Carena, J.R. Ellis, A. Pilaftsis and C.E.M. Wagner, *Renormalization-group-improved effective potential for the MSSM Higgs sector with explicit CP-violation*, *Nucl. Phys.* **B 586** (2000) 92 [hep-ph/0003180]; *Higgs-boson pole masses in the MSSM with explicit CP-violation*, *Nucl. Phys.* **B 625** (2002) 345 [hep-ph/0111245].

[19] P.G. Harris et al., *New experimental limit on the electric dipole moment of the neutron*, *Phys. Rev. Lett.* **82** (1999) 904;
S.K. Lamoreaux and R. Golub, *Comment on 'new experimental limit on the electric dipole moment of the neutron'*, *Phys. Rev.* **D 61** (2000) 051301 [hep-ph/9907282].

[20] M. Leurer, Y. Nir and N. Seiberg, *Mass matrix models: the sequel*, *Nucl. Phys.* **B 420** (1994) 468 [hep-ph/9310320];
G.R. Dvali, G.F. Giudice and A. Pomarol, *The μ-problem in theories with gauge-mediated supersymmetry breaking*, *Nucl. Phys.* **B 478** (1996) 31 [hep-ph/9603238].

[21] D. Suematsu, *mu problem in an extended gauge mediation supersymmetry breaking*, hep-ph/0308007.

[22] T. Kobayashi, D. Suematsu and Y. Yamagishi, *Gauge coupling unification due to nonuniversal soft supersymmetry breaking*, *Phys. Lett.* **B 329** (1994) 27 [hep-ph/9403330].

[23] E. de Rafael, *The muon g-2 revisited*, hep-ph/0208251.

[24] T. Falk, K.A. Olive, M. Pospelov and R. Roiban, *MSSM predictions for the electric dipole moment of the ^{199}Hg atom*, *Nucl. Phys.* **B 560** (1999) 3 [hep-ph/9904393].

[25] D. Suematsu and H. Tsuchida, in preparation.

[26] For a review, see for example, L. Randall and C. Csáki, *The doublet-triplet splitting problem and higgses as pseudogoldstone bosons*, hep-ph/9508208.

[27] M. Carena, J.R. Ellis, A. Pilaftsis and C.E.M. Wagner, *Renormalization-group-improved effective potential for the MSSM Higgs sector with explicit CP-violation*, *Nucl. Phys.* **B 586** (2000) 92 [hep-ph/0003180]; *Higgs-boson pole masses in the MSSM with explicit CP-violation*, *Nucl. Phys.* **B 625** (2002) 345 [hep-ph/0111245];
J.S. Lee et al., *Cpsuperh: a computational tool for Higgs phenomenology in the minimal supersymmetric standard model with explicit CP-violation*, *Comput. Phys. Commun.* **156** (2004) 283 [hep-ph/0307377].

[28] E. Witten, *Deconstruction, G_2 holonomy and doublet-triplet splitting*, hep-ph/0201018.

[29] N. Arkani-Hamed, A.G. Cohen and H. Georgi, *(De)constructing dimensions*, *Phys. Rev. Lett.* **86** (2001) 4757 [hep-th/0104005]; *Electroweak symmetry breaking from dimensional deconstruction*, *Phys. Lett.* **B 513** (2001) 232 [hep-ph/0105239].

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